Graph Algorithms with MapReduce

S4230

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Credits:
- MapReduce: The Definitive Guide, Tom White
- Jeffery Dean and Sanjay Chemawat. *MapRecuce*
- Jimmy Lin and Chris Dyer. *Data Intensive Text Processing with MapReduce*

Today's Topics

- Introduction to graph algorithms and graph representations
- Single Source Shortest Path (SSSP) problem
  - Refresher: Dijkstra's algorithm
  - Breadth-First Search with MapReduce
- PageRank

What’s a graph?

- G = (V,E), where
  - V represents the set of vertices (nodes)
  - E represents the set of edges (links)
  - Both vertices and edges may contain additional information
- Different types of graphs:
  - Directed vs. undirected edges
  - Presence or absence of cycles
- Graphs are everywhere:
  - Hyperlink structure of the Web
  - Physical structure of computers on the Internet
  - Interstate highway system
  - Social networks

Some Graph Problems

- Finding shortest paths
  - Routing Internet traffic and UPS trucks
- Finding minimum spanning trees
  - Telco laying down optical fiber
- Finding Max Flow
  - Airline scheduling
- Identify “special” nodes and communities
  - Breaking up terrorist cells, spread of avian flu
- Bipartite matching
  - Monster.com, Match.com
- PageRank, HITS, EdgeRank

Graphs and MapReduce

- Graph algorithms typically involve:
  - Performing computation at each node
  - Processing node-specific data, edge-specific data, and link structure
  - Traversing the graph in some manner
- Key questions:
  - How do you represent graph data in MapReduce?
  - How do you traverse a graph in MapReduce?

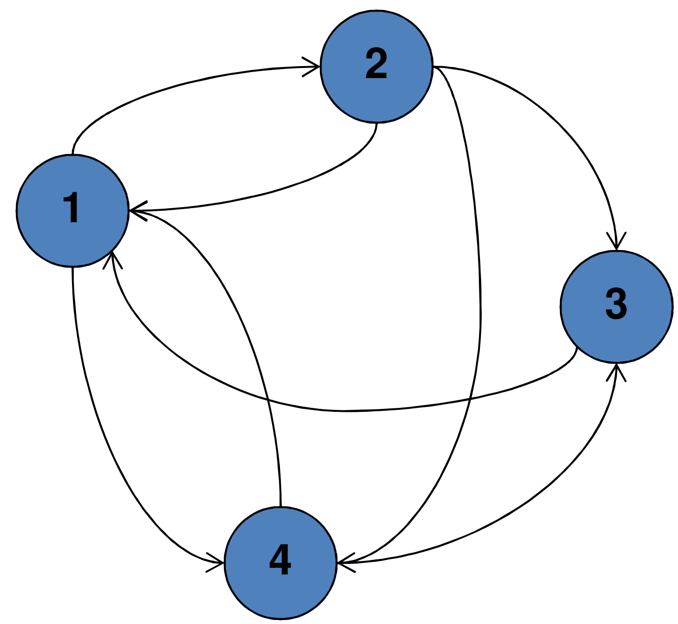
Representation Graphs

- G = (V, E)
  - A poor representation for computational purposes
- Two common representations
  - Adjacency matrix
  - Adjacency list

Adjacency Matrices

- Represent a graph as an n x n square matrix M
  - $n = |V|$
  - $M_{ij} = 1$ means a link from node i to j

	1	**2**	**3**	**4**
1	0	1	0	1
2	1	0	1	1
3	0	1	0	0
4	1	0	1	0



Adjacency Matrices: Critique

- Advantages:
  - Naturally encapsulates iteration over nodes
  - Rows and columns correspond to inlinks and outlinks
- Disadvantages:
  - Lots of zeros for sparse matrices
  - Lots of wasted space

Adjacency Lists

- Take adjacency matrices… and throw away all the zeros
- Represent only outlinks from a node

	1	**2**	**3**	**4**
1	0	1	0	1
2	1	0	1	1
3	0	1	0	0
4	1	0	1	0

1: 2, 4
2: 1, 3, 4
3: 1
4: 1, 3

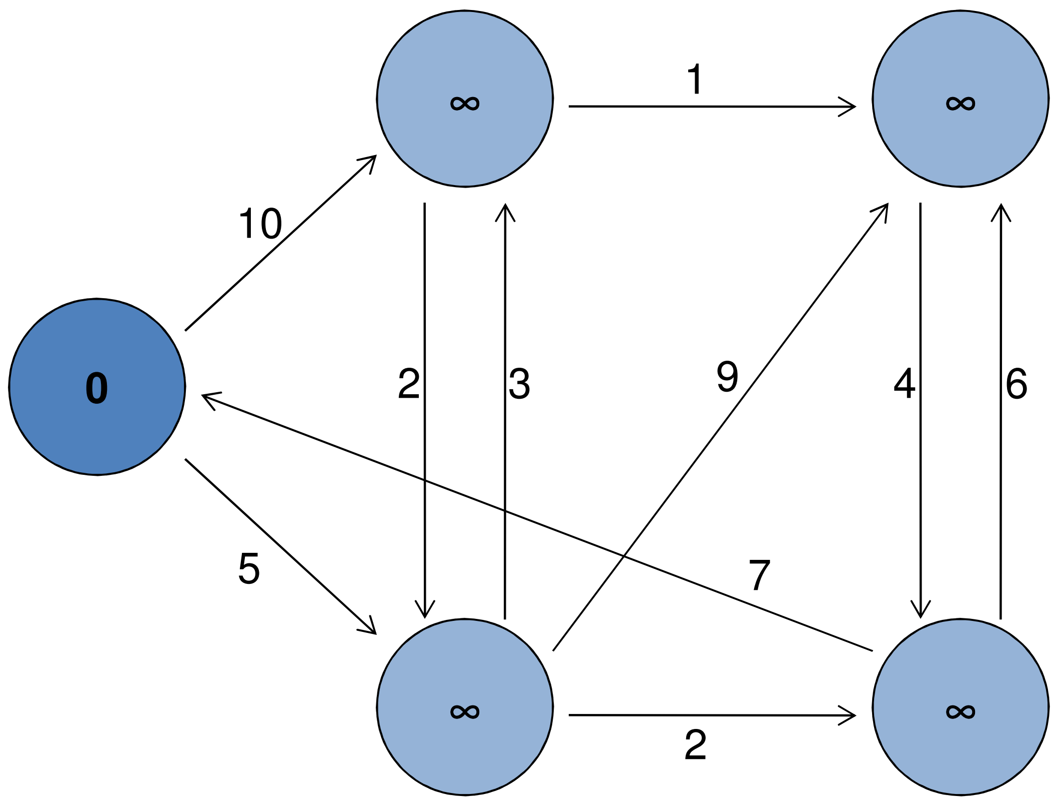
Adjacency Lists: Critique

- Advantages:
  - Much more compact representation
  - Easy to compute over out-links
  - Graph structure can be broken up and distributed
- Disadvantages:
  - More difficult to compute over in-links

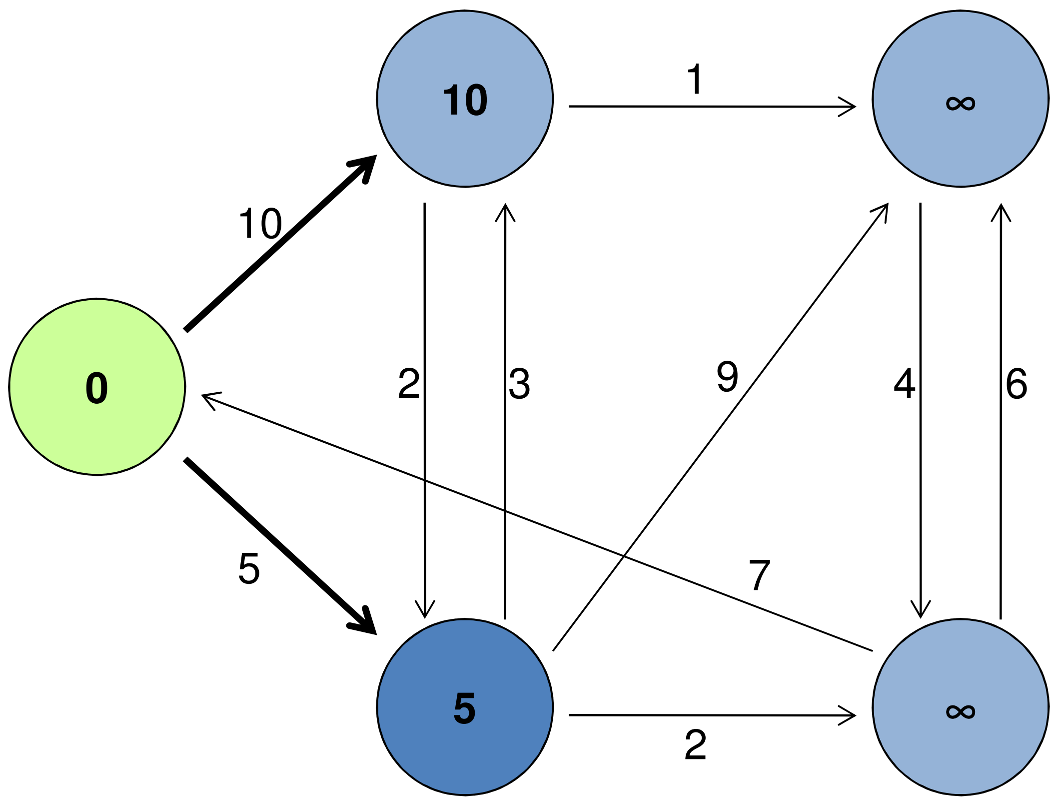
Single Source Shortest Path

- **Problem:** find shortest path from a source node to one or more target nodes
- First, a refresher: Dijkstra's Algorithm

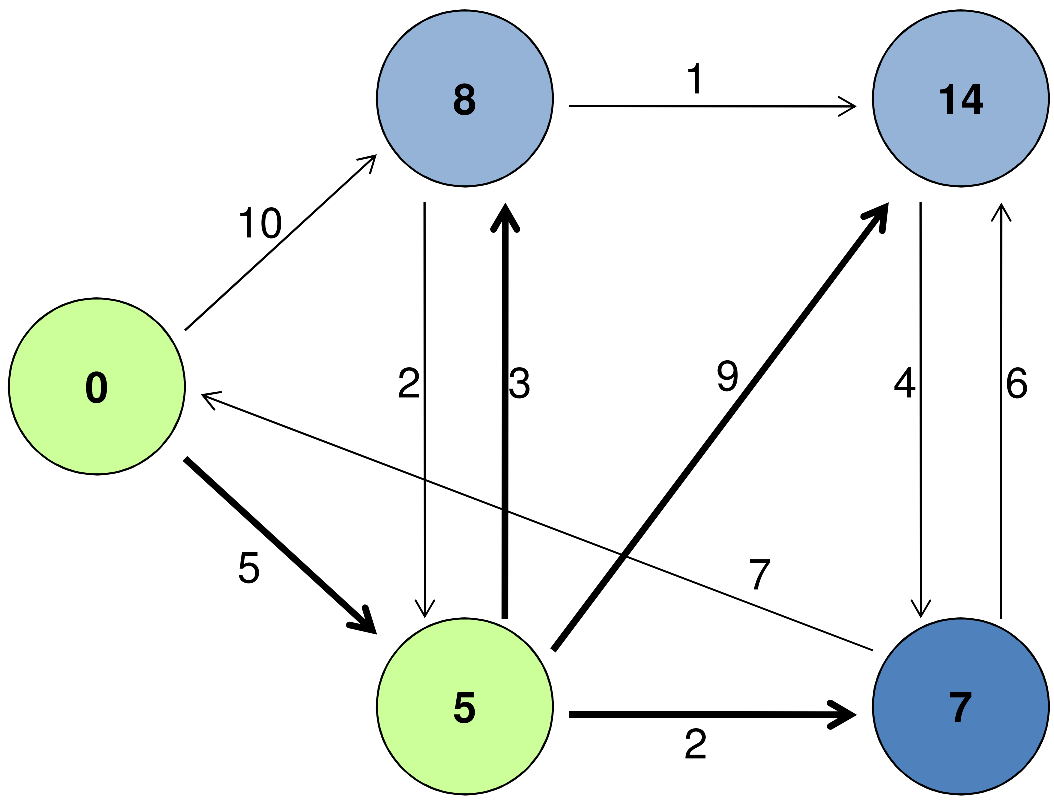
Dijkstra’s Algorithm Example



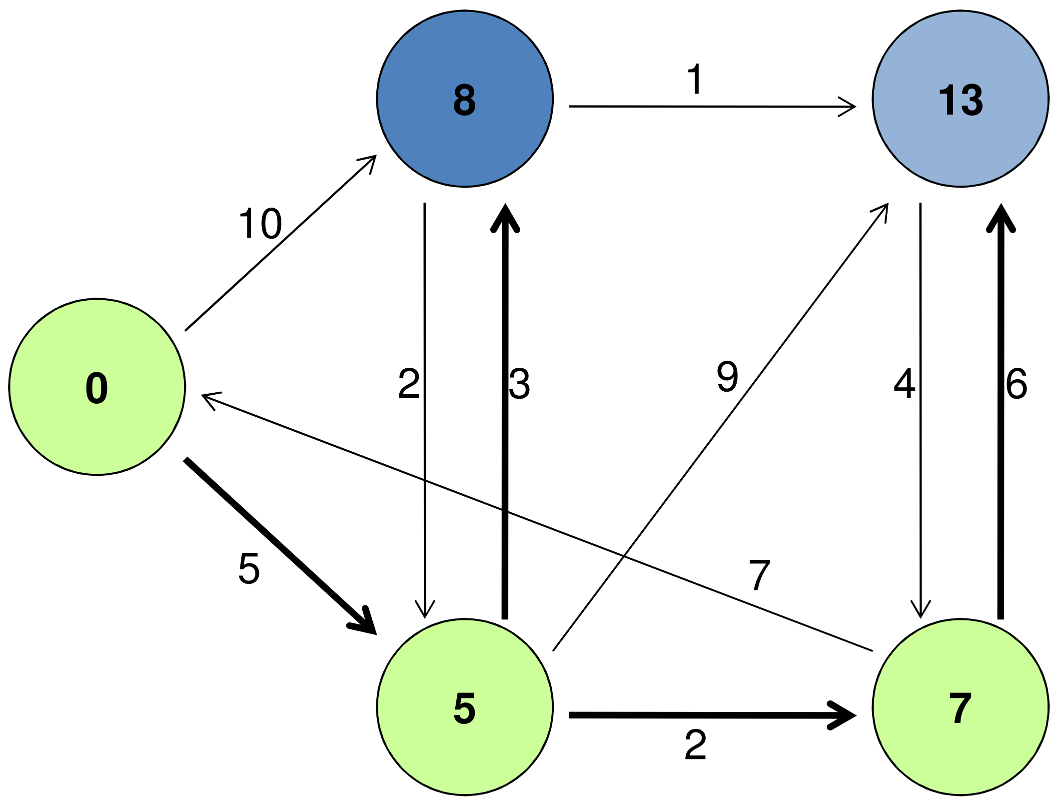
Dijkstra’s Algorithm Example



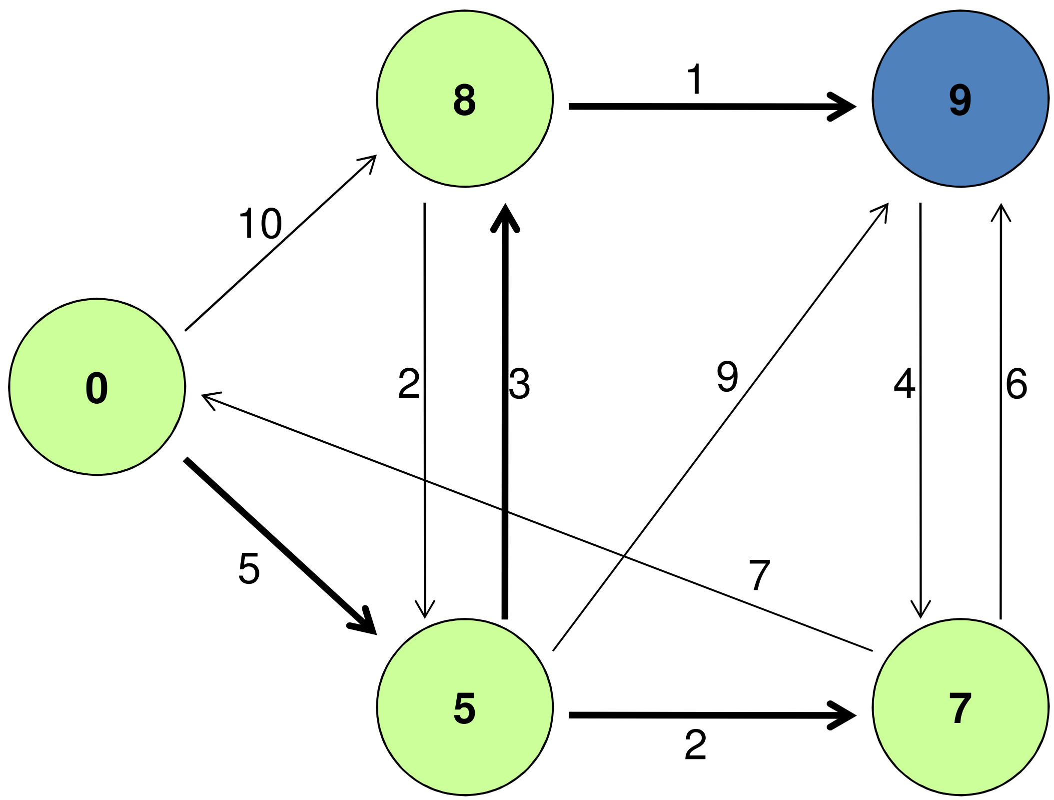
Dijkstra’s Algorithm Example



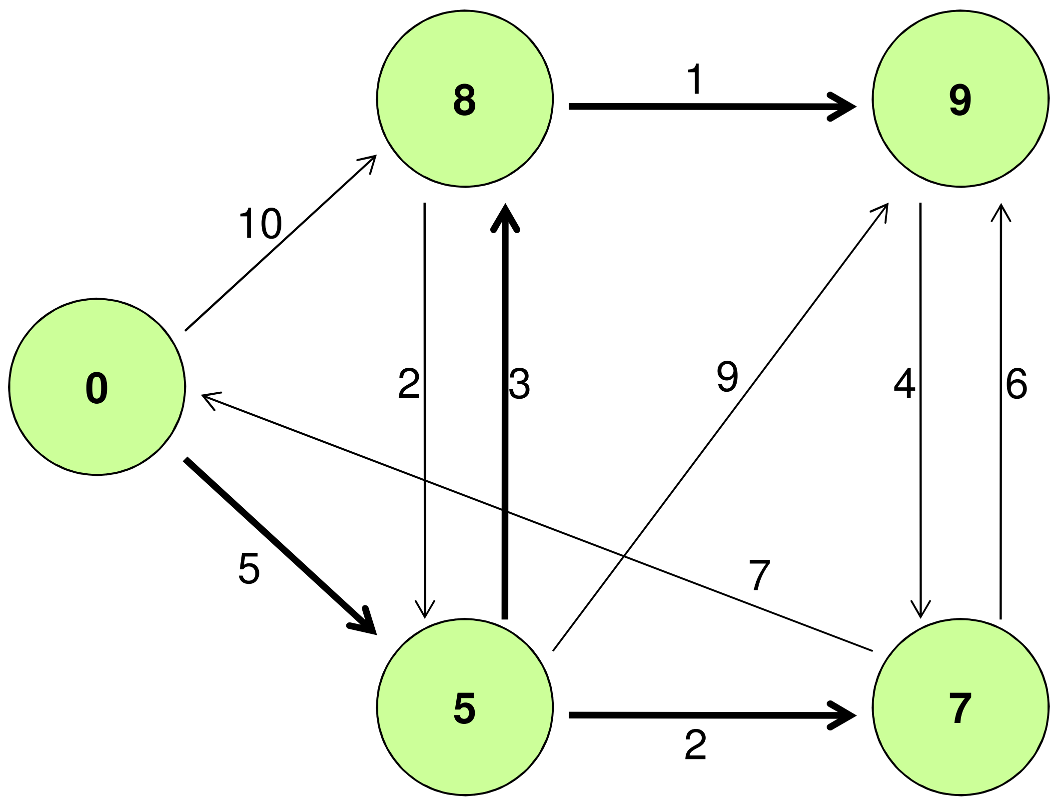
Dijkstra's Algorithm Example



Dijkstra’s Algorithm Example



Dijkstra's Algorithm Example



Single Source Shortest Path

- **Problem:** find shortest path from a source node to one or more target nodes
- Single processor machine: Dijkstra’s Algorithm
- MapReduce: parallel Breadth-First Search (BFS)

Finding the Shortest Path

- First, consider equal edge weights
- Solution to the problem can be defined inductively
- Here's the intuition:
  - DistanceTo(startNode) = 0
  - For all nodes *n* directly reachable from startNode, DistanceTo(*n*) = 1
  - For all nodes *n* reachable from some other set of nodes S, DistanceTo(n) = 1 + min(DistanceTo(m), $m \in S$)

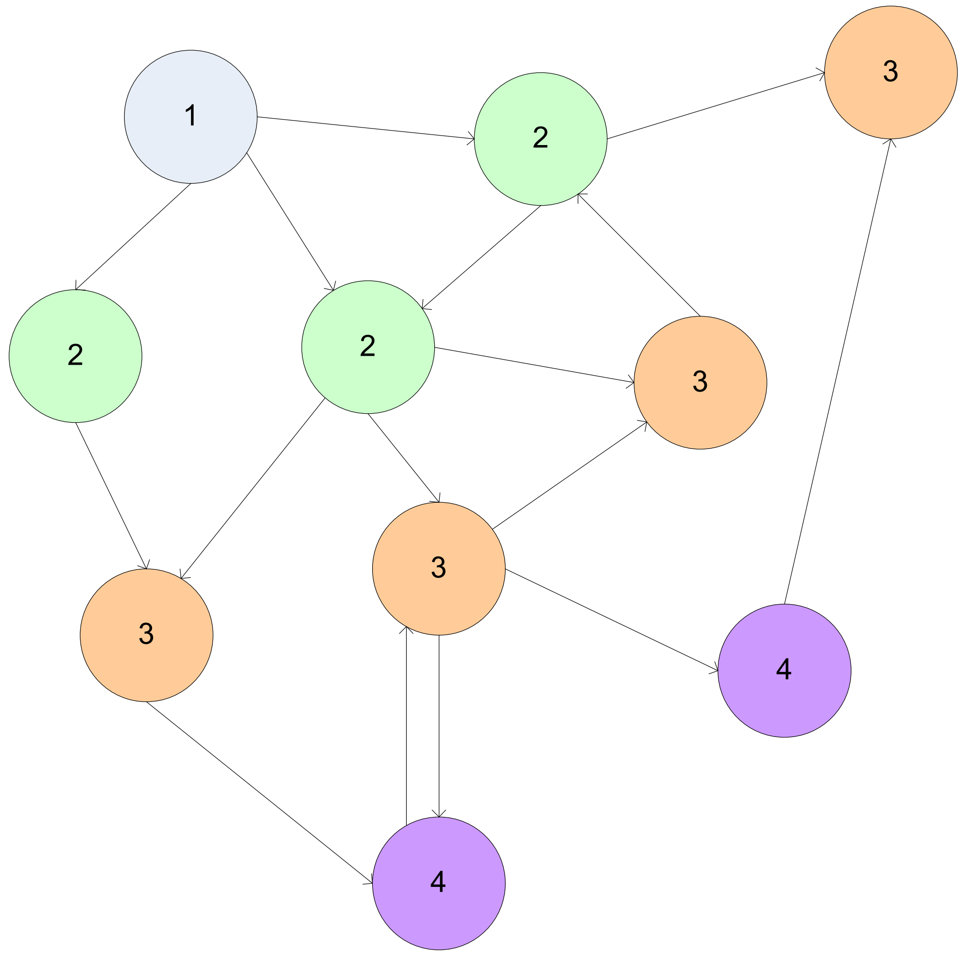
From Intuition to Algorithm

- A map task receives
  - Key: node *n*
  - Value: D (distance from start), points-to (list of nodes reachable from *n*)
- $\forall p \in$ points-to: emit (p, D+1)
- The reduce task gathers possible distances to a given *p* and selects the minimum one

Multiple Iterations Needed

- This MapReduce task advances the "known frontier" by one hop
  - Subsequent iterations include more reachable nodes as frontier advances
  - Multiple iterations are needed to explore entire graph
  - Feed output back into the same MapReduce task
- Preserving graph structure:
  - Problem: Where did the points-to list go?
  - **Solution: Mapper emits (*n*, points-to) as well**

Visualizing Parallel BFS



Termination

- Does the algorithm ever terminate?
  - Eventually, all nodes will be discovered, all edges will be considered (in a connected graph)
- When do we stop?

Weighted Edges

- Now add positive weights to the edges
- Simple change: points-to list in map task includes a weight *w* for each pointed-to node
  - emit (p, D+w_p) instead of (p, D+1) for each node p
- Does this ever terminate?
  - Yes! Eventually, no better distances will be found. When distance is the same, we stop
  - Mapper should emit (n, D) to ensure that “current distance” is carried into the reducer

Graph

- a: b, c
- b: c, d
- c:
- d:
- e:

```
Mapper ( a, (0, (b,c)))
    Emit( b, (1, (c,d)))
    Emit( c, (1, ()))
    …
Reducer ( b, (1, (c,d)))
    (b,1)<-min(b,1)
    output(b, (1, (c,d)))
Reducer ( c, (1, ())
    (c,1)<- min(c,1)
    output(c, (1, ()))
```

```
Mapper ( b, (1, (c,d)))
    Emit( c, (2, ()))
    Emit( d, (2, ()))
    …
Reducer ( c, (2, ()))
    (c,1)<- min(c,2)
    // no output
Reducer ( d, (2, ()))
    // no output
    (d,1)<- min(d,2)
    // no output
```

Comparison to Dijkstra

- Dijkstra’s algorithm is more efficient
  - At any step it only pursues edges from the minimum-cost path inside the frontier
- MapReduce explores all paths in parallel
  - Divide and conquer
  - **Throw more hardware at the problem!**

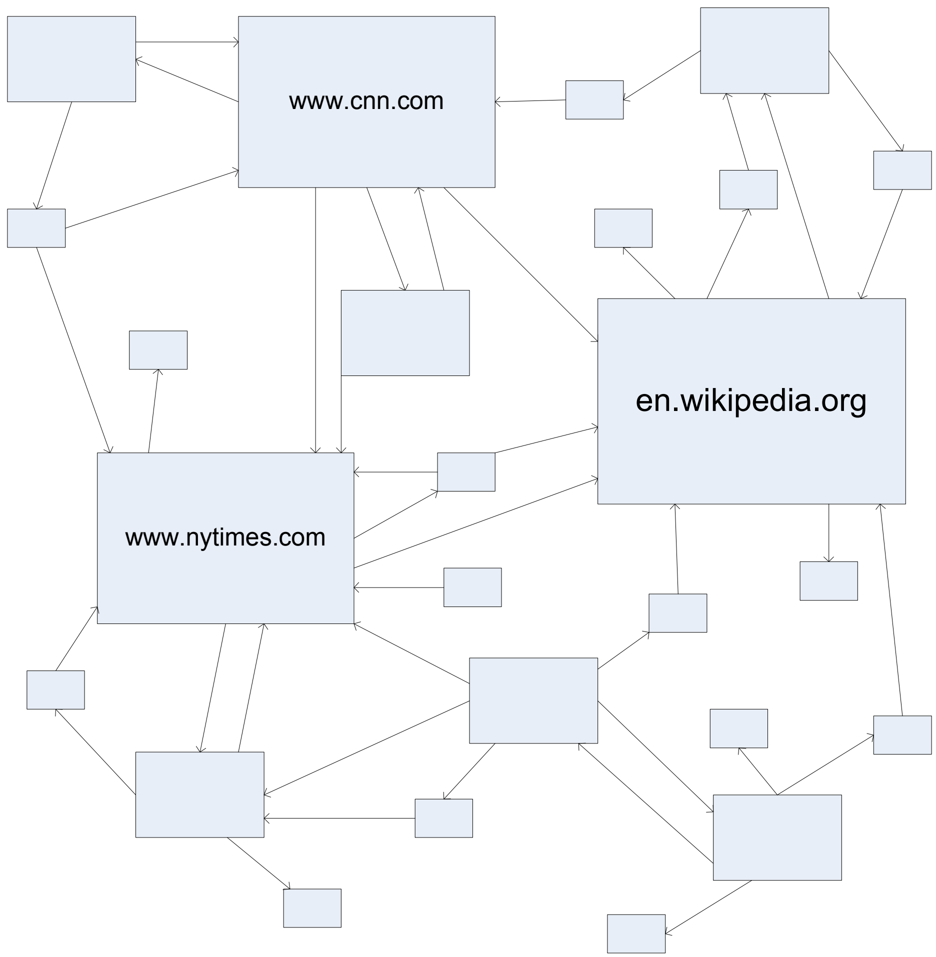
General Approach

- MapReduce is adept at manipulating graphs
  - Store graphs as adjacency lists
- Graph algorithms with MapReduce:
  - Each map task receives a node and its outlinks
  - Map task compute some function of the link structure, emits value with target as the key
  - Reduce task collects keys (target nodes) and aggregates
- Iterate multiple MapReduce cycles until some termination condition:
  - **Remember to “pass” graph structure from one iteration to next**

Random Walks Over the Web

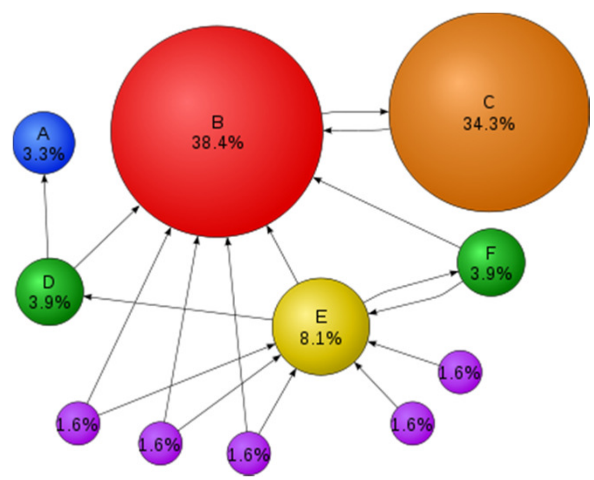
- Model:
  - User starts at a random Web page
  - User randomly clicks on links, surfing from page to page
- What’s the amount of time that will be spent on any given page?
- This is PageRank

PageRank: Visually



PageRank

- Initially developed at Stanford University by Google founders, Larry Page and Sergey Brin, in 1995.
- Program implemented by Google to rank any type of recursive "documents" using MapReduce.
- Led to a functional prototype named Google in 1998.
- Still provides an important function for Google's web search tools.



PageRank

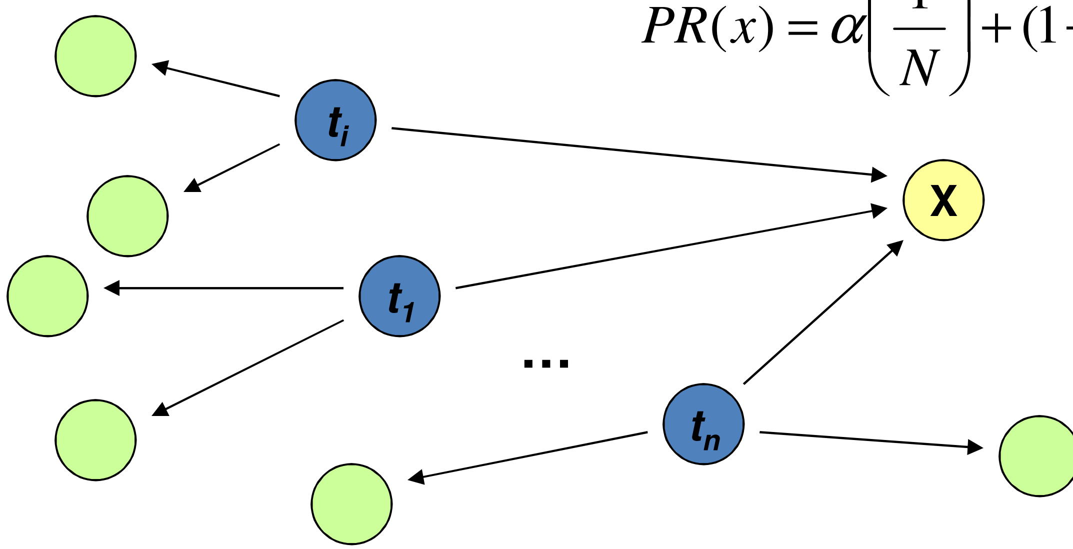
- Assume a small universe of four web pages: **A**, **B**, **C** and **D**. The initial approximation of PageRank would be evenly divided between these four documents.
- Each document would begin with an estimated PageRank of 0.25.
- If the only links in the system were from pages **B**, **C**, and **D** to **A**, each link would transfer 0.25 PageRank to **A** upon the next iteration, for a total of 0.75.

$$PR(A) = PR(B) + PR(C) + PR(D).$$

PageRank: Defined

- Given page *x* with in-bound links $t_1 \ldots t_n$, where
  - $C(t)$ is the out-degree of t
  - α is probability of random jump
  - N is the total number of nodes in the graph
- We can define PageRank as:

$$PR(x) = \alpha\left(\frac{1}{N}\right) + (1-\alpha)\sum_{i=1}^{n}\frac{PR(t_i)}{C(t_i)}$$



PageRank

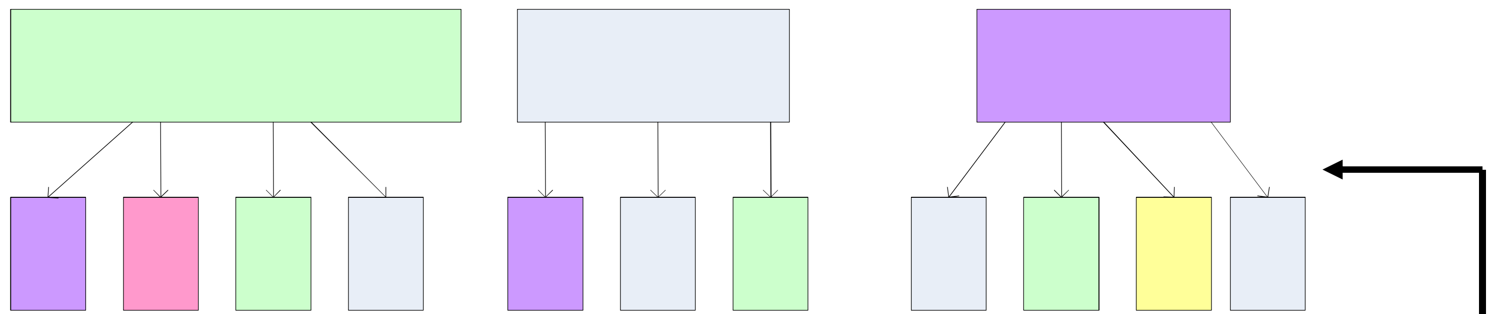
- Simulates a “random-surfer”
- Begins with pair (URL, list-of-URLs)
- Maps to (URL, (PR, list-of-URLs))
- Maps again taking above data, and for each *u in list-of-URLs* returns *(u,* PR/|list-of-URLs|), as well as (*u, new-list-of-URLs)*
- Reduce receives (URL, list-of-URLs), and many (URL, value) pairs and calculates (URL, (new-PR, list-of-URLs))

Computing PageRank

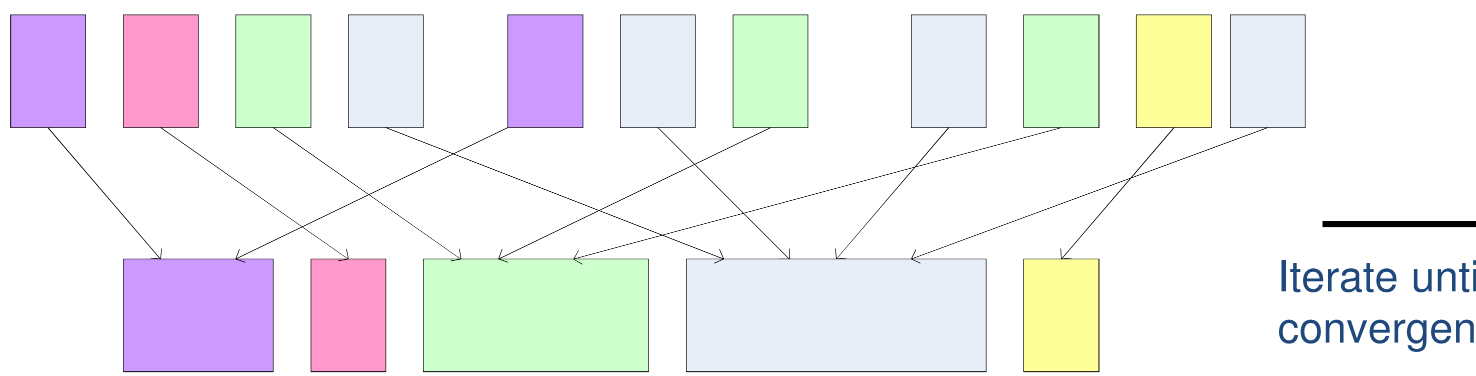
- Properties of PageRank
  - Can be computed iteratively
  - Effects at each iteration is local
- Sketch of algorithm:
  - Start with seed PR_i values
  - Each page distributes PR_i “credit” to all pages it links to
  - Each target page adds up “credit” from multiple in-bound links to compute PR_{i+1}
  - Iterate until values converge

PageRank in MapReduce

Map: distribute PageRank "credit" to link targets



Reduce: gather up PageRank "credit" from multiple sources to compute new PageRank value



Iterate until convergence

...

PageRank: Issues

- Is PageRank guaranteed to converge? How quickly?
- What is the "correct" value of α, and how sensitive is the algorithm to it?
- What about dangling links?
- How do you know when to stop?